Floer Cohomology and Arc Spaces

Mark McLean

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- Very roughly, the **arc space** of a singularity is the space of holomorphic maps from the unit disk passing through that singularity. Actually this is called the **short arc space**.
- We will be only interested in jets of such maps.
- However, my personal opinion is that the entire arc space is important if one wishes to study other more complicated Floer groups.

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- The **Milnor map** is defined to be the symplectic fibration

$$\frac{f}{|f|} : S_\epsilon - f^{-1}(0) \longrightarrow S^1.$$

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$$\phi_f : M_f \longrightarrow M_f$$

is the monodromy of the Milnor fibration around S^1.

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  - M is a compact manifold with boundary.
  - $d\theta$ is a symplectic form.
  - The unique vector field X_θ satisfying $i_{X_\theta} d\theta = \theta$ points outwards along ∂M.

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- **Fact:** $f^{-1}(0)$ is a contact submanifold of S_ϵ. This is called the **link** of f.

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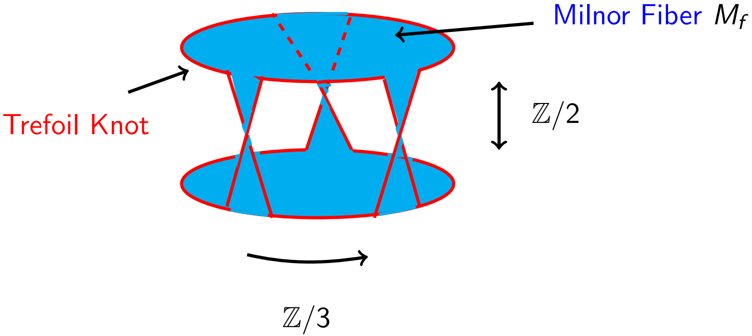

ϕ_f generates $\mathbb{Z}/6 = \mathbb{Z}/2 \times \mathbb{Z}/3$ action
(modulo some boundary rotation factor)

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- *Technical remark:* Grading is given by minus Conley-Zehnder index, with trivialization induced by $\mathbb{C}^n$.

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- In other words, Floer cohomology is an invariant of the link as a contact submanifold.

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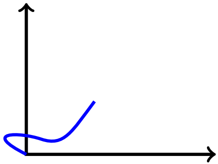

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- Morally, it is the space of d-jets of holomorphic maps $\mathbb{D} \longrightarrow \mathbb{C}$ whose boundary ‘wraps’ around the singularity d times.

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Theorem (M, In progress). The conjecture above is true in general.

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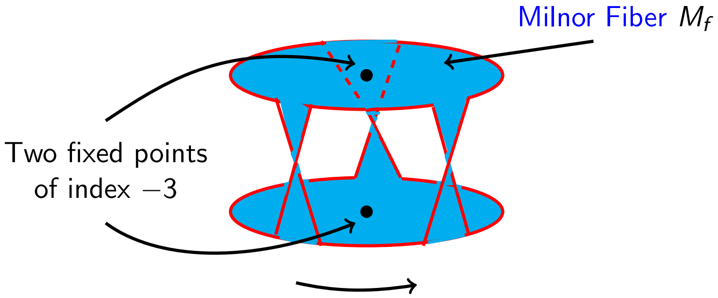

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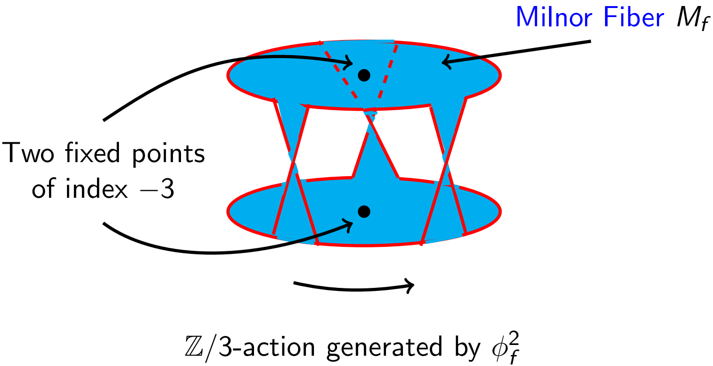

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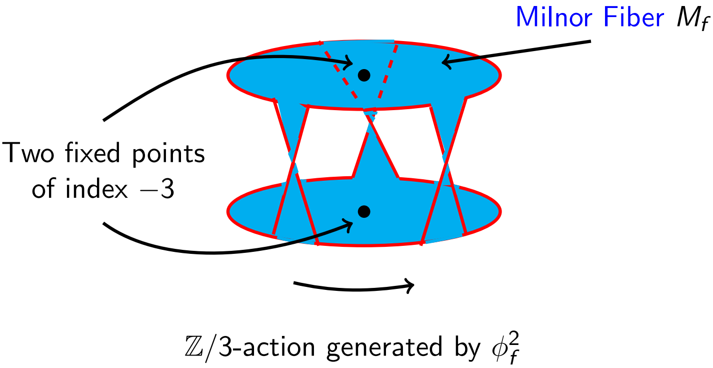

- $HF^*(\phi_f^2) = \mathbb{Z} \oplus \mathbb{Z}$ if $* = 2$ and 0 otherwise. Hence the Conjecture is true in this example.

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- This map is similar in spirit to the log PSS map defined by Ganatra and Pomerleano.

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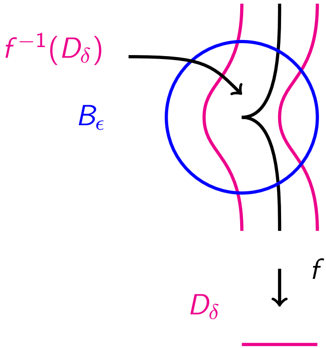

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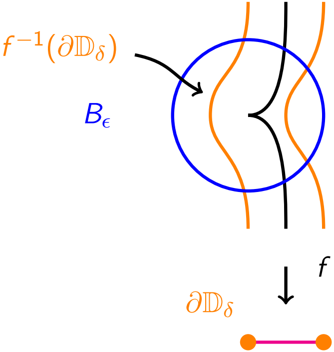

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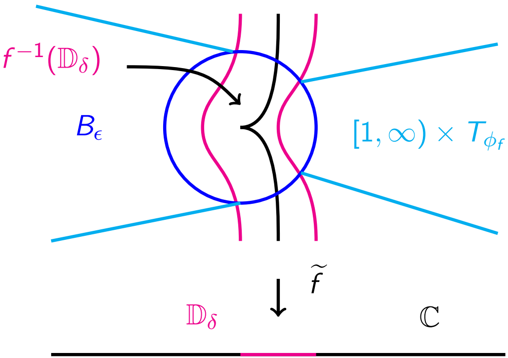

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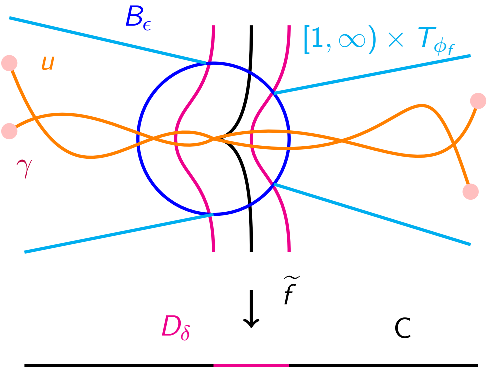

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A standard PSS style gluing and compactness argument ensures that ev is a chain map.

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However in this talk we will suppress this detail.

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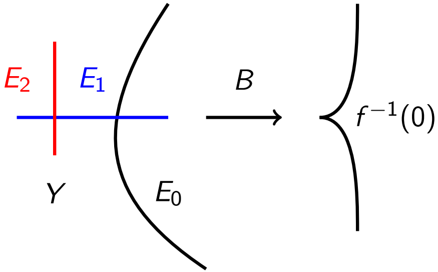

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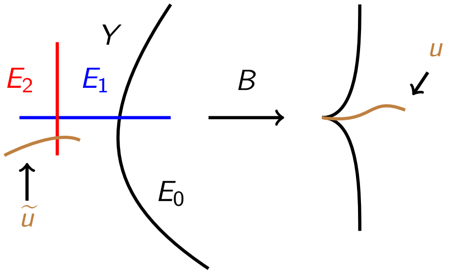

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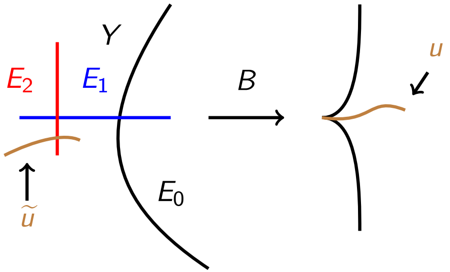

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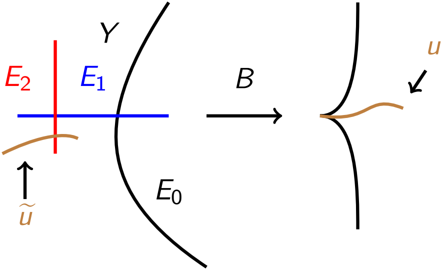

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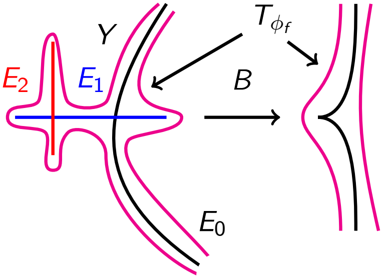

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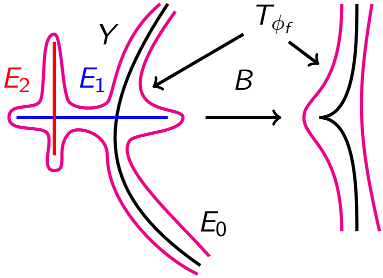

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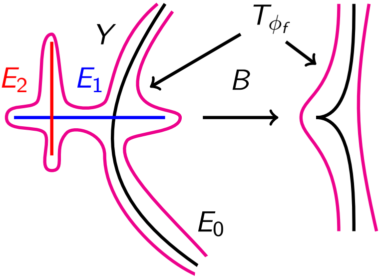

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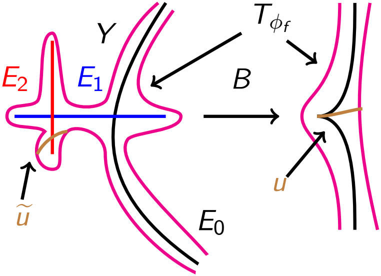
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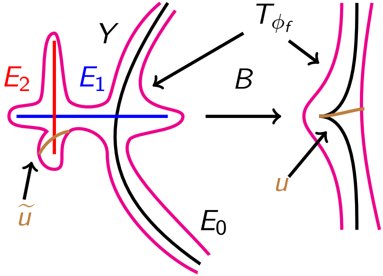

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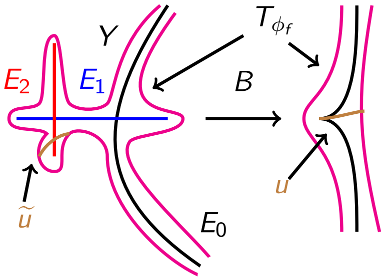

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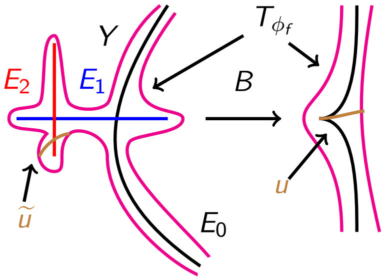

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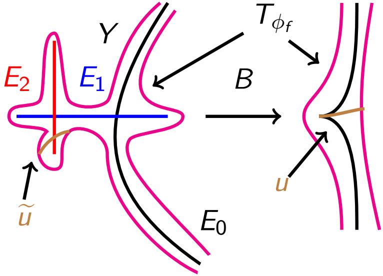

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- Ganatra and Pomerleano constructed a spectral sequence computing symplectic cohomology of an affine variety from a smooth normal crossing compactification. What happens if the compactification is no longer smooth normal crossing? The hope is that one can build the E^1 page from various spaces of (low energy) arcs.

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